

Numerical Study on Interaction Between Two Bubbles Rising Side by Side in CMC Solution^{*}

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Abstract A numerical simulation was performed to investigate the interaction of two bubbles rising side by side in shear-thinning fluid using volume of fluid (VOF) method coupled with continuous surface force (CSF) method. By considering rheological characteristics of fluid, this approach was able to accurately capture the deformation of bubble interface, and validated by comparing with the experimental results. The rising of bubble pairs with different configurations, including horizontal alignment and oblique alignment, was simulated by the method. The influences of the bubble initial distance and the bubble alignment were studied by analyzing the bubble deformation, rising paths and flow fields surrounding bubbles. The results indicate that within certain initial bubble spacing of $S^* = 3.3$ ($S^* = S_l/D$, S_l initial distance between bubbles, and D bubble diameter), the dynamic interaction between two bubbles aligned horizontally shows repulsive effect that decreases with the increase of initial bubble spacing, but weakens to certain degree by the shear-thinning properties of fluid. However, the interaction between two bubbles aligned obliquely presents a repulsive effect for the small angle involved but an attractive impact for the large one, which is yet strengthened by the rheological characteristics of fluid.

Keywords bubble pairs, interaction, shear-thinning, volume of fluid method, rheological characteristics

1 INTRODUCTION

Rising of bubbles in non-Newtonian fluids has received considerable attention in the literature owing to its inherent scientific interest and significance in industrial application, such as chemical, biochemical, environmental, petrochemical processes [1, 2]. In typical equipment which employs bubbly flow, such as bubbles column and bioreactor, bubble-bubble interaction is frequently encountered and will cause remarkable changes in the flow fields and the bubble dynamics in non-Newtonian fluids. Not only the shape, velocity and the track of bubble are affected by the bubble interaction, but also coalescence or breakup can take place *via* bubble collisions, consequently leading to a crucial impact on the hydrodynamics and the mass and heat transfer between gas-liquid phases. For an in-line bubble pairs rising in non-Newtonian fluids under the buoyancy force, previous investigations on various aspects have been reported theoretically and experimentally. In contrast, knowledge of the dynamics of bubble pairs aligned horizontally, which is more relevant to real industrial processes, still remain to be considered due to the inherently complex nature of non-Newtonian fluids. Thus, adequate knowledge for interaction dynamics of a pair of bubbles rising side by side in non-Newtonian fluids is essential to the understanding of gas-liquid equipment design and an overall insight into bubble coalescence mechanism.

Compared to lots of research on single bubble [3], much less is known about the bubble pairs behaviors rising in non-Newtonian fluids. However, most of the

experimental and numerical investigations were focused on the wake formed behind the leading bubble and its effect on trailing bubble, as well as its difference from that in Newtonian fluids. Unlike the dragging effect of the wake in Newtonian fluids, a negative wake in non-Newtonian fluids was firstly observed to push the liquid away from the bubble by Hassagar [4]. Subsequently, by means of laser-doppler anemometry (LDA), Bisgaard and Hassagar [5] concluded that a negative wake was induced by elasticity. This peculiar phenomenon has also been observed for spheres falling in viscoelastic liquids [6]. Furthermore, Frank and Li [7, 8] found the coexistence of three distinct zones around bubbles rising in polyacrylamide (PAM) solutions. Sousa *et al.* [9] found that for the most concentrated carboxymethyl cellulose (CMC) solution, a negative wake was also formed behind the Taylor bubbles. Lin and Lin [10] has proposed that for two in-line bubbles, the acceleration of the trailing bubble to the leading bubble owes to the dragging force caused by the negative pressure and the shear-thinning effect in addition to the pushing force caused by the viscoelastic effect. However, few attempts have been made to investigate experimentally the interaction between bubble pairs rising side by side in non-Newtonian fluids. Fan *et al.* [11] focused the rise and interaction between two parallel rising bubbles by analyzing the velocity field around bubbles using particle image velocimetry (PIV), and found that within certain distance between two bubbles, the interaction between two neighboring bubbles changed from mutual repulsion to attraction with the decrease of the angle of the line of linking two bubble centers to the vertical direction.

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Besides the experimental techniques, numerical simulation method has become an increasingly popular approach for the study of bubble dynamics due to its high capability to obtain some essential physical information. By a finite difference method based simplified marker and cell (SMAC) implicit solver on a staggered grid, Kishorea *et al.* [12] proposed a simple correlation for the total drag coefficient for a single sphere sedimenting in power-law liquids. Tsamopoulos *et al.* [13] simulated the rising and deformation of a bubble in a viscoelastic fluid *via* the mixed finite element method coupled with a quasi-elliptic mesh generation scheme. Singh and Denn [14] carried out numerically the interaction of two and three bubbles rising in a Bingham fluid using a regularization finite-element method. By rheological simulation and birefringence visualization, Li [15] and Li *et al.* [16] proved that in-line interactions are governed by a dynamical competition between the creation and relaxation of stresses in non-Newtonian fluids. With rapid development of numerical method, the motion and deformation of bubble or drop in non-Newtonian fluids have been explored respectively using level set method [17], volume of fluid [18, 19], first principle method [20, 21] and Lattice Boltzmann method [22]. Recently, the motion of deformable gas bubbles rising freely in quiescent shear-thinning solutions within a wide Reynolds number range has been simulated by an improved level set method [23]. Nevertheless, these experimental and numerical studies cited above focused frequently on the hydrodynamic interaction of in-line bubble pairs in non-Newtonian flows. In contrast, few numerical investigations of the rising and interaction of bubble pairs with horizontal alignment in non-Newtonian fluids have been reported except for the work of Vélez-Cordero *et al.* [24]. According to Ref. [24], bubbles rising in the fluids experienced so-called drafting-kissing-tumbling process.

Although considerable progress of numerical research on the motion behavior of bubble pairs rising in Newtonian fluids has been achieved, the research about this issue in non-Newtonian fluids, especially for two bubbles aligned horizontally, remains still insufficient in the literature. The scanty of previous works was carried out by 2D calculation [24]. However, the complex dynamic interface cannot be reflected by 2D simulations. For example, the bubble asymmetric deformation or spiral rise in a bubble column should be computed by 3D simulations. In this work, by taking into account the rheological characteristics of shear-thinning fluid, a 3D simulation based on CFD approach was performed to study the rising process of two bubbles in shear-thinning fluid combining volume of fluid (VOF) method with continuous surface force (CSF) method, and the interaction between two bubbles aligned at various arrangement was discussed.

2 SIMULATION

The modeling approach selected in the simulation

was Volume-of-Fluid (VOF) [25] method which is a volume-tracking method used to model free surfaces. The rising of bubble pairs side by side driven by buoyancy in a quiescent shear-thinning liquid was considered for modeling purpose with the following assumptions: (1) the fluids in both phases were incompressible and immiscible; (2) the two-phase flow was laminar and isothermal. The 3D computational domain is sketched in Fig. 1.

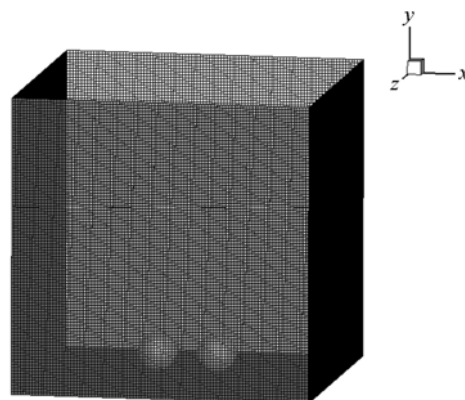


Figure 1 Schematic diagram of solution domain for numerical simulation

2.1 Governing equations

In this method, a particular phase ' q ' is identified by its volume fraction (α_q) in a computational cell. If $\alpha_q = 1$, the cell is full of the q th (secondary) phase; if $\alpha_q = 0$, the cell is full of the p th (primary) phase and if $0 < \alpha_q < 1$, the cell contains the interface between the p th and q th phases. The equation of conservation of mass and momentum governing both the liquid and gas phase are given by Eqs. (1) and (2). Tracking of the interface(s) between the phases is implemented by the solution of continuity equation for the volume fraction of one of the phases as given by the following Eq. (3).

$$\nabla \cdot \mathbf{U} = 0 \quad (1)$$

$$\rho(\alpha) \left(\frac{\partial \mathbf{U}}{\partial t} + \nabla(\mathbf{U}\mathbf{U}) \right) = -\nabla p + \nabla[2\mu(\alpha)\mathbf{D}] + \mathbf{F}_s + \rho(\alpha)\mathbf{g} \quad (2)$$

$$\frac{\partial \alpha}{\partial t} + \mathbf{U} \cdot \nabla \alpha = 0 \quad (3)$$

where $\mathbf{U}$ is the velocity vector, p the pressure, μ dynamics viscosity coefficient, $\mathbf{F}_s$ volume force due to surface tension, $\rho(\alpha)$ the density and $\mathbf{g}$ the gravity. Rate-of-deformation tensor, $\mathbf{D}$ is

$$\mathbf{D} = \frac{1}{2} [\nabla \mathbf{U} + (\nabla \mathbf{U})^T] \quad (4)$$

When a computational cell is not entirely occupied by one phase, mixture properties estimated by Eqs. (5)

and (6) are used to solve Eq. (2):

$$\rho(\alpha) = \rho_l(\alpha) + \rho_g(1 - \alpha) \quad (5)$$

$$\mu(\alpha) = \mu_l(\alpha) + \mu_g(1 - \alpha) \quad (6)$$

where the subscripts *l* and *g* are used for liquid phase and gas phase, respectively.

2.2 Momentum source term considering interfacial tension

The interfacial tension along the interface between fluids must be taken into account due to its key effects on the bubble motion and deformation, since the small curvature of gas-liquid interface tends to bring the larger additional pressure in these flow conditions. In the paper, the CSF model [26] was used to compute the surface tension force for the cells containing the gas-liquid interface:

$$F_s = \sigma \frac{\rho \kappa \nabla \alpha_l}{\frac{1}{2}(\rho_l + \rho_g)} \quad (7)$$

where, σ the surface tension, local surface curvature, κ , is calculated as follows [26]:

$$\kappa = 0(\nabla \hat{n}) = \frac{1}{|\mathbf{n}|} \left[\frac{\mathbf{n}}{|\mathbf{n}|} \nabla \cdot \mathbf{n} - (\nabla \mathbf{n}) \right] \quad (8)$$

2.3 Constitutive equation for shear-thinning fluid

In this paper, according to the intermediate region of shear rate, the constitutive equation of a shear-thinning fluid could be expressed by the Power-law model [27]:

$$\mu(\alpha) = K \dot{\gamma}^{(n-1)} \quad (9)$$

Once the local shear rate, $\dot{\gamma}$, has been calculated at each nodal point in the domain, the apparent viscosity μ can be reconstructed, whereas the local viscosity changed instantaneously with the velocity vector by the following formula [21]:

$$\dot{\gamma} = \sqrt{2(\mathbf{D} : \mathbf{D})} \quad (10)$$

Therefore, the viscosity of a shear-thinning non-Newtonian fluid was obtained by combining Eqs. (4), (9) and (10), which were carried out through compiling a user-defined-function (UDF) program.

2.4 Solution domain, boundary conditions and numerical schemes

In the present work, a cubic 3D solution domain (52 mm×52 mm×52 mm) with uniform structured grid was used. The geometry for bubble pairs close to the bottom wall was built and the mesh was created using software Gambit 2.3 as shown in Fig. 1. Note that a

grid independence certification was performed by simulating the bubbles rising under the same conditions ($t = 0.28$ s, $D = 6.0$ mm, 0.8% CMC solution and $S_1 = 10.0$ mm) using different sized grids to ensure the accuracy of numerical results. The typical effects of mesh size on predicted terminal shapes of bubble pairs under the same conditions are given in Fig. 2. The different cell size 0.4 mm, 0.5 mm, 0.6 mm and 0.7 mm were tested for calculation. It is found that the simulation is accurate enough with the 0.4 mm, 0.5 mm resolutions, and considering computational time required, the mesh size of 0.5 mm (corresponding 1124, 864 cells) was adopted for subsequent simulations. Consequently, there exist 104 cells in *x*, *y* and *z* direction and 12 cells for one bubble diameter.

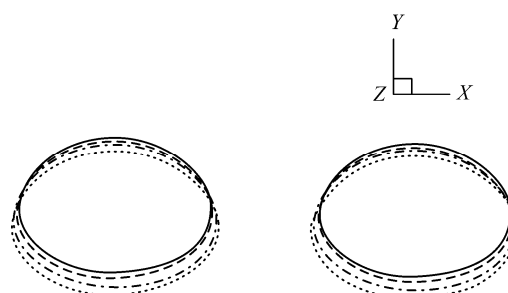


Figure 2 Simulation results of two bubbles rising in quiescent liquid at different computing meshes ($t = 0.28$ s, $D = 6.0$ mm, 0.8% CMC solution and $S_1 = 10.0$ mm)
mesh size/mm: — 0.4; - - - 0.5; - · - · - 0.6; ····· 0.7

In the initial calculation, two spherical bubbles with diameter D were set symmetrically along the vertical axis of symmetry in the computational domain. Both the liquid and the bubble were assumed to be stationary at the initial state. The boundary conditions given in Fluent 6.3 were the pressure one at outlet for top surface, the stationary wall with no slip for side surfaces, and the gravity effect was considered in the whole process.

Model Eqs. (1), (2), and (3) given above were solved using the segregated unsteady solver in FLUENT. Discrimination terms were set as follows: the PRESTO! scheme for the pressure term, second order up-wind for the momentum equation, and the Geo-Reconstruction scheme for the volume fraction. The pressure implicit with splitting of operator (PISO) algorithm was used for the pressure-velocity coupling in the momentum equation. The relative error between two successive iterations was specified by using a convergence criterion of 1×10^{-3} for each scaled residual components, and the time steps used in the simulations are usually 1.0×10^{-5} s.

3 RESULTS AND DISCUSSION

3.1 Comparison with experimental results

To validate the numerical method and to offer a reference for the investigation of bubble interaction,

the buoyant rising process of twin bubbles is first simulated. Especially, the experiment under the same condition was carried out in plexiglas square tank with two identical polished orifices (inside diameter 1.0 mm) at tank bottom. Note that the orifice spacing was adjusted to 10.0 mm, the bubble volume was set accurately by using two syringes, thus, the two bubble rising process can be observed and acquired by digital camera. Fig. 3 shows respectively the numerical results and experimental observations of rising sequence for bubble pairs in 0.8% (by mass) CMC aqueous solution with bubble diameter $D=6.0$ mm. The liquid properties ($\rho = 1.0102 \text{ g}\cdot\text{cm}^{-3}$; $\sigma = 0.06804 \text{ N}\cdot\text{m}^{-1}$; $K = 0.127 \text{ Pa}\cdot\text{s}^n$, $n = 0.904$) coupled with shear-thinning effect yield a Morton number Mo from 4.5×10^{-4} to 6.0×10^{-4} , and the Eötvös number Eu is 5.2 for both bubbles. The Reynold number Re corresponding to the terminal velocity is 12.3. From the view of whole rising process, it is clearly seen that due to the increasingly weakening of the interaction of the adjacent bubble, both simulated bubble and measured bubble take on egg shape in initial period and then evolves gradually into ellipsoidal shapes with the flat bottoms in the final period. Meanwhile, the quantitative comparison of bubble characteristics, including aspect ratio, rising distance and bubble spacing, are presented in Fig. 4. Nevertheless, the reasonable agreement is found between the simulated bubbles shape, trajectory at every rising moment and the photographs taken in the experiments. Thus, it implies that by considering rheological characteristics of shear-thinning fluid, the numerical technique based on the VOF method can accurately predict the shape and path of two bubbles throughout the rising process under the present conditions, and can be qualified for the rising of bubble pairs with various alignments.

3.2 Bubble-bubble interaction: horizontal alignment

Under low Reynolds number, the interaction between a pair of bubbles is mainly influenced by the center-to-center distance and their relative configuration, as shown in Fig. 5, where β is the angle between the centerline of bubbles and the horizontal direction. Here, the interaction between two horizontally aligned bubbles is studied with three various initial distance $S^* = 1.7, 3.3, 5.0$ with $\beta = 0$, where $S^* = S_i/D$, and S_i being the initial distance between the bubble pairs, and D the bubble diameter.

Figure 6 shows the simulated results on the rising process of abreast bubble pairs with three initial distance with $Eu = 5.2$ and $Re < 14.1$. The results reveal that the path, shape as well as velocity experience a significant variation with the increase of the distance between the centers of two bubbles. For $S^* = 1.7$, bubbles deform gradually from spherical shape to ellipsoidal shape and then to ellipsoidal shapes with the flat bottoms. Both bubbles start to deviate vertically from the middle point between the two bubbles, and finally recover their horizontal poses with terminal shape. Consequently, the distance between two bubbles increases first fast and then slow as shown in Fig. 7. It is indicated that the horizontal repulsion between them plays a key role in their rising process in shear-thinning fluid within simulated Reynolds numbers. The reason has been attributed to either the dominant viscous force due to the vertical flow developed around the bubbles for small value of Re [28], or the dominant potential force due to the sufficiently thin boundary layer developed on the bubble surface for large Re [29].

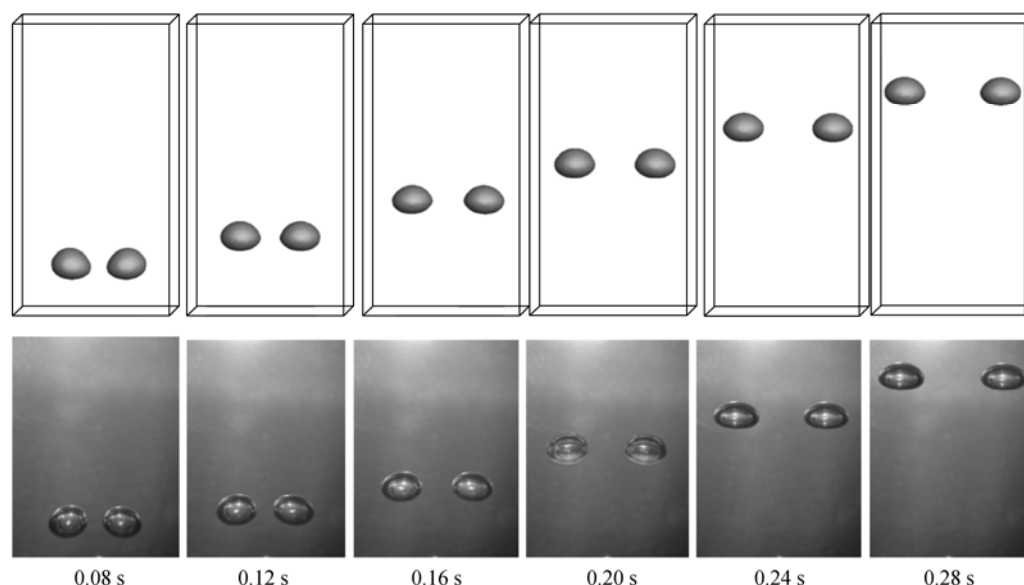


Figure 3 A series of instantaneous state of rising bubble pairs in 0.8% CMC aqueous solution for (up): numerical results and (low): experimental measurements ($S_i/D = 1.7$, $D = 6.0$ mm)

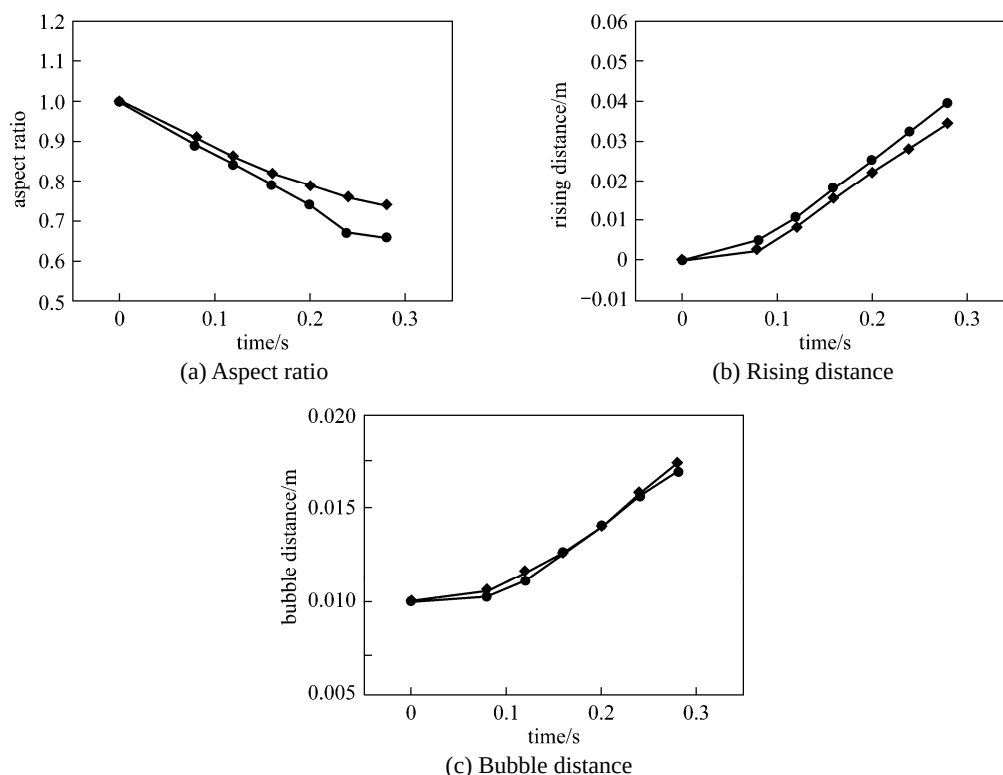


Figure 4 Comparison between simulation and experiment for aspect ratio, rising distance and bubble distance

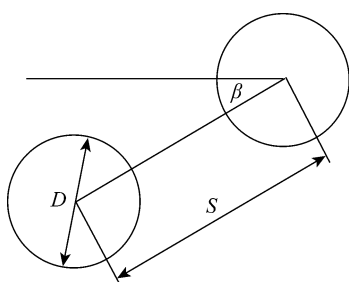


Figure 5 Schematics of the configuration of a bubble pair

In contrast, for $S^* = 3.3$, only a minute repulsive effect between two bubbles is observed: the bubble deformation takes place mainly within the beginning stage (< 0.16 s), as shown in Fig. 6 (b), however, the distance between bubbles gets rise slowly in the later rising stage, as shown in Fig. 7. However for $S^* = 5.0$, the bubble-bubble interaction is hardly found in simulated condition: both bubbles rise in a straight trajectory without horizontal motion, like single bubbles as illustrated in Fig. 6 (c). Besides, it is worth noting that the bubble velocity in rising process increases with the distance between bubbles, in other words, the vertical motion of bubbles with small distance is weakened by their repulsive motion in horizontal direction. So it is reasonable to conclude that bubble-bubble interaction exhibits a repulsive effect, which decreases with the distance between bubbles under simulated conditions.

Furthermore, a series of typical flow fields around two-bubble pairs in above distance condition is

compared at the meantime and the results are presented in Fig. 8. It is demonstrated that the wakes behind each bubble are formed since bubbles start to rise due to buoyancy. For small bubbles distance, for example $S^* = 1.7$ or 3.3 , both wakes formed behind two side-by-side bubbles will encounter each other, leading to their deflections away from the symmetrical axis, and consequently both bubbles are pushed away in the deflected directions. However, for $S^* = 5.0$, no deflection of both wakes behind bubbles is observed, and the wakes do not impact each, implying that bubble-bubble interaction does not occur anymore. Especially, the wakes induced by both bubbles are influenced little due to larger distance between them, hence resulting in no obvious change of the horizontal repulsion between bubbles, even by virtue of the shear-thinning effect of fluid. This truth was also attributed to the dominant role of the wake instability on the motion of a pair of bubbles with changing direction [30].

3.3 Bubble-bubble interaction: the effect of β

The interaction between two bubbles with an oblique configuration is investigated by considering their relevant position, in terms of β . The contours of the bubble surface at different time for two initial angles β : 17° and 31° are respectively illustrated in Fig. 9, with the identically horizontal distance of $S^* = 1.7$. It is manifested that for $Eo = 5.2$ and $Re < 14.7$, when the configuration angle β increases, the interaction

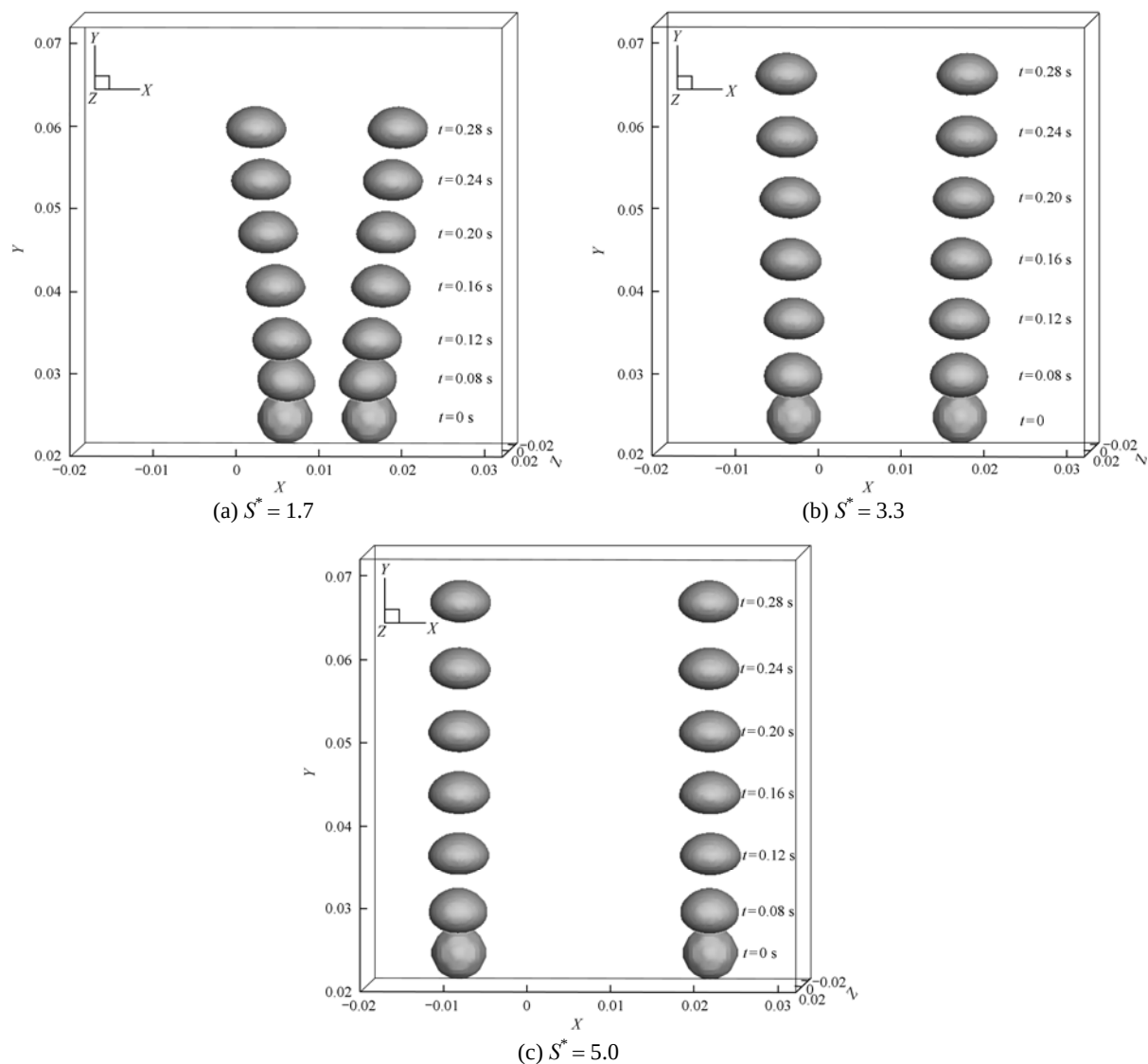


Figure 6 Simulation results of rising process of horizontal bubble pairs ($D = 6$ mm) with various distance for $S^* = 1.7, 3.3$ and 5.0

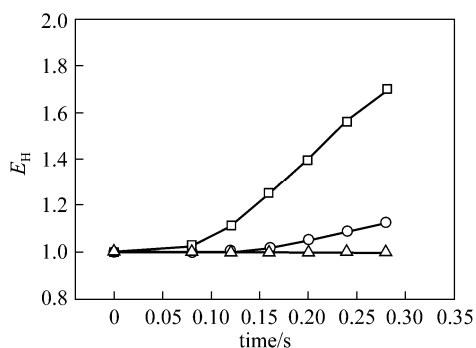


Figure 7 Bubble distance variation with time during rising process of bubble pairs ($D = 6$ mm) (variation ratio of bubble distance $E_H = S/S_0$, S the practical distance between bubble pairs)
 S^* : $\square$ 1.7; $\circ$ 3.3; $\triangle$ 5.0

between two bubbles alter from repulsion to attraction as the bubbles rise in the domain shown in Figs. 9 (a) and 9 (b), respectively. For small angle ($\beta = 17^\circ$), when the bubbles begin moving, the lower bubble is pulled up by fluid flow induced by the wake behind the upper one, conversely leading to a pushed-away effect on the upper one. Thus, the both bubble shapes deform significantly and their rising paths deviate gradually from the original vertical line, resulting in increase of the bubble distance. However, For large angle ($\beta = 31^\circ$), the lower bubble approach slowly and eventually is drawn by the wake of upper one, then the horizontal attraction between the bubble pairs becomes obvious and they gradually gets closer after the time of 0.12 s. Besides, it is noted that in case of large angle, the lower bubble eventually evolves into egg shape with its flattened bottom due to the attraction of wake behind upper one, whereas significant clockwise

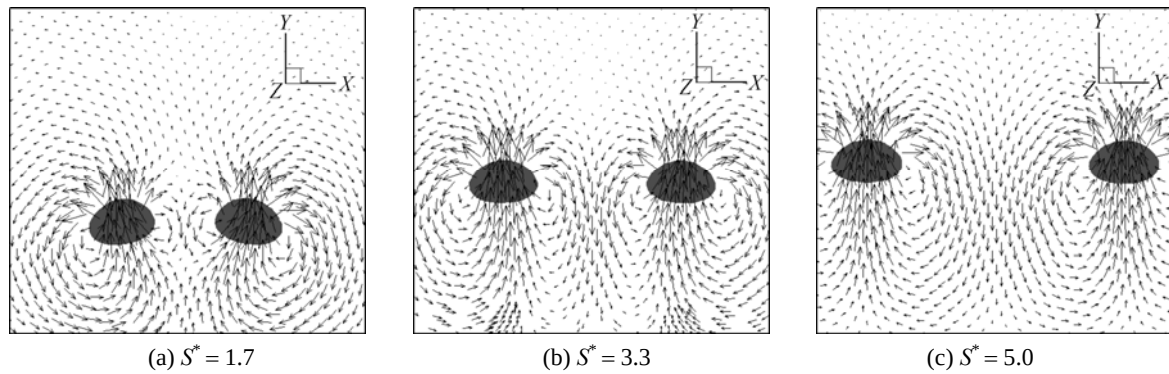


Figure 8 Instantaneous flow field around rising bubble pairs ($D = 6$ mm) at the time of 0.16 s with $S^* = 1.7, 3.3$ and 5.0

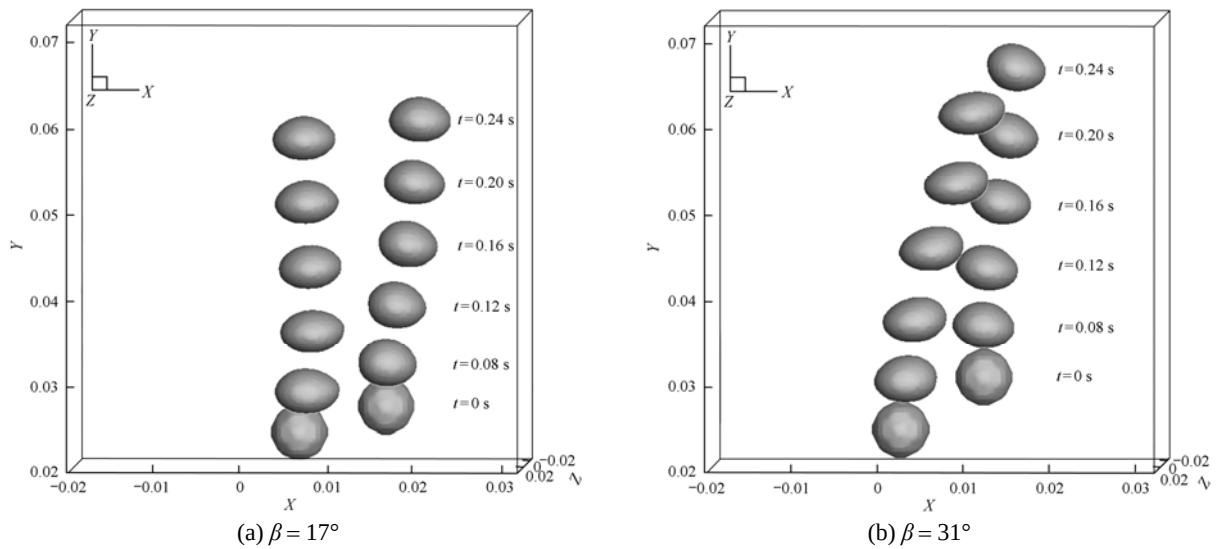


Figure 9 Simulation results of rising process of two oblique bubbles with $S^* = 1.7$ and $D = 6$ mm for β : 17° and 31°

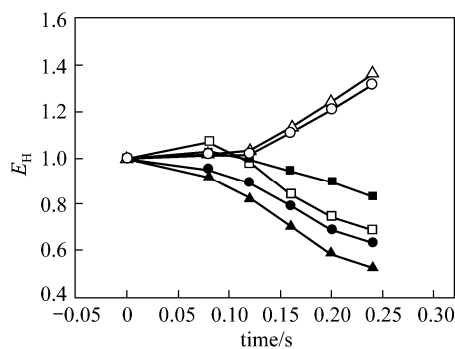


Figure 10 Variation ratio of nondimensional distance between bubbles with time during their rise ($S^* = 1.7$)
 $\beta = 17^\circ$: $\square$ vertical distance; $\triangle$ horizontal distance; $\circ$ total distance
 $\beta = 31^\circ$: $\blacksquare$ vertical distance; $\blacktriangle$ horizontal distance; $\bullet$ total distance

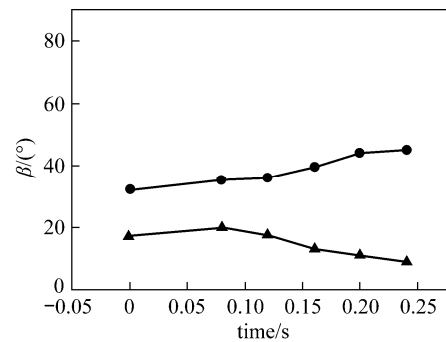


Figure 11 Variation of configuration angle during the rise of bubble pairs initially aligned at 17° and 31° ($S^* = 1.7$)
 β : $\bullet$ 17° ; $\blacktriangle$ 31°

rotation occurs in the upper one with asymmetric ellipsoidal shape owing to the pressing effect of lower on. Moreover, the vertical rising velocity of both bubbles aligned with large angle is higher than that with small angle. The reason can be attributed to the deduction of viscosity of local region, produced by shear-thinning

effect of the wake behind the upper bubble, rather than repel each other in Newtonian fluid [24].

The variation ratio of bubble distance, defined as E_H , is applied to describe the relative motion between the two bubbles, and the results is shown respectively in Figs. 10 and 11. Note that E_H includes two vertical

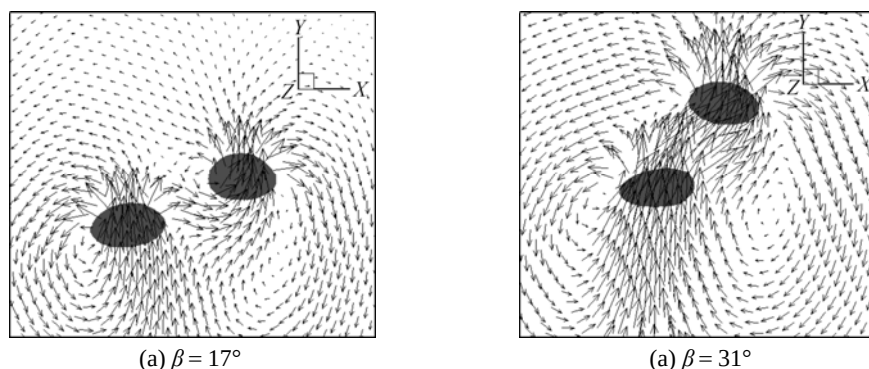


Figure 12 Instantaneous flow field around two bubble pairs $S^* = 1.7$ at time of 0.20 s with $\beta = 17^\circ$ and $\beta = 31^\circ$

and horizontal ratios. Firstly, for the vertical distance, the ratio increases at the maximum value about 1.1, and then decreases steadily in case of $\beta = 17^\circ$. However, for $\beta = 31^\circ$, the curve switches from increase to decrease, up to ending value of 0.84. Secondly, for horizontal distance, the ratio increases slowly followed by rapidly for $\beta = 17^\circ$, whereas decreases with the similar trend for $\beta = 31^\circ$. It is indicated that for small angle, there always exists a repulsive impact between bubbles in both directions, whereas for large angle, an attraction effect between bubbles is often displayed in both directions except for their initial rise stage. Especially, the horizontal interaction becomes crucial to the effect of “push” or “pull” between bubbles since the variation ratio curves of both horizontal distance and total distance have the same trend during the computed domain, consequently causing different variation of β as shown in Fig. 11.

Figure 12 presents the typical flow field around bubble pairs at time of 0.20 s. It is revealed that under small angle condition, the upward flow of the liquid between bubbles collide each other and will push both bubbles far away. In contrast, for large angle, the lower bubble is inclined to be caught by the wake of upper one, which results in the acceleration of lower one toward upper one due to the shear-thinning effect of CMC solution, consequently the distance between them decreases with the rising process. Nevertheless, there might exist an equilibrium angle at which the repulsive and attractive interaction are balance, which is essentially a competitive result between repulsion in horizontal direction and attraction in vertical direction.

4 CONCLUSIONS

The interaction between twin bubbles aligned with various configuration has been numerically investigated by adopting volume of fluid (VOF) method combining continuous surface force (CSF) method. By considering rheological characteristics of shear-thinning fluid, it has been demonstrated that the use of this method can satisfactorily simulate the interaction and deformation process of twin bubbles rising in a quiescent shear-thinning fluids in low Re number by comparing

with experimental measurements. The influences of the initial distance between two bubbles, as well as the angle between the centerline of bubbles and the horizontal direction on the interaction of bubbles, were discussed respectively by analyzing the bubble deformation, rising paths as well as flow fields surrounding bubbles.

The results indicated that for two rising bubbles with horizontal configuration, the interaction between them shows repulsive effect, which decrease with the initial center-to-center distance of bubbles despite reduction by the shear-thinning properties of fluid. The reason lies in that for the small initial distance between bubbles, both wakes formed behind bubbles act each other heavily, resulting in a strong push-away effect on both ones. However, for two oblique bubbles, there might exist an equilibrium angle at which the repulsive and attractive interactions are balance. Above the equilibrium value, the dynamics interaction between two bubbles presents repulsive effect, whereas shows attractive impact for below the value under this simulated condition.

NOMENCLATURE

D	bubble diameter, m
$\mathbf{D}$	rate-of-deformation tensor
E_H	variation ratio of bubble distance (S/S_i)
Eu	Eötvös number [$Eu = D^2 g(\rho_l - \rho_g)/\sigma$]
$\mathbf{F}_s$	volume force, $\text{N} \cdot \text{m}^{-3}$
$\mathbf{g}$	gravitational acceleration, $\text{m} \cdot \text{s}^{-2}$
K	consistency coefficient, $\text{Pa} \cdot \text{s}^n$
Mo	Morton number ($Mo = g \mu_l^4 / \rho_l \sigma^3$)
$\mathbf{n}$	normal vector
$\hat{\mathbf{n}}$	unit normal vector
p	pressure, Pa
Re	Reynolds number ($Re = D u \rho / \mu_l$)
S	actual distance between bubbles, m
S_i	initial distance between bubbles, m
S^*	dimensionless distance between bubbles (S_i/D)
t	time, s
$\mathbf{U}$	velocity vector
α	volume fraction function
β	angle between the centerline of bubbles and the horizontal direction
$\dot{\gamma}$	local shear rate, s^{-1}
κ	local surface curvature

μ	apparent viscosity, Pa·s
ρ	density of fluid, kg·m ⁻³
σ	surface tension, N·m ⁻¹

Subscripts

g	gas
l	liquid
p	the p th phase
q	the q th phase

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